ONE APPROACH TO THE CONSTRUCTION OF CONSTITUTIVE EQUATIONS IN CREEP THEORY

V. P. Golub UDC 539.376

The problem of constructing adequate constitutive equations to describe creep is one of the most complex problems in continuum mechanics. The problem of describing deformation to fracture is especially important. It is presently solved either on the basis of a dimensionless damage parameter (see [1-4], for example) or through the use of a damage function that can be identified with unit fracture work [5]. The main difficulty in these approaches lies in obtaining reliable estimates for the damage criterion. In the present investigation, we propose an approach to construction of constitutive creep equations which makes it possible to describe all stages of the process without the use of a damage function. The approach being proposed here also makes it possible to allow for the initial ductility properties of the material.

1. Introduction. With regard to the solution of problems involving construction of constitutive equations, the concept that is most frequently employed in creep theory is that of the "mechanical equation of state"

$$\Phi\left(\sigma_i,\ \varepsilon_i^c,\ \dot{\varepsilon}_i^c,\ t, T\right)=0, \tag{1.1}$$

where Φ is an empirical function; σ_i, $\varepsilon_i{}^c$, $\dot{\sigma}_i{}^c$ are the intensities of the stresses, strains, and creep rates, respectively; t is time; T is temperature.

The specific form of the function Φ is found from tests conducted in uniaxial tension. The form of the function is most often found using the similarity principle, making it possible to represent Φ in the form of a product of functions. Also used are the concepts of aging and strain-hardening (as set forth by Soderberg and Davenport, respectively) and Rabotnov's concept of damage [1, 3-6]. However, this approach usually requires refinement before it can adequately describe an actual process. This has to do with the need to experimentally substantiate the "single creep curve" hypothesis and establish the laws governing the effect of damage on creep rate and the accumulation of creep damages.

Satisfaction of the similarity condition [1, 4, 5, 7] is of fundamental importance. All of the existing forms of Eqs. (1.1) have been found using the assumption that similitude of creep processes is observed in a coordinate system in which the time t is one of the variables and the stresses and temperature are parameters. However, such similitude is absent for the steady-state and accelerated stages of creep in most cases [1, 3, 7].

More promising is an approach in which similarity of creep processes is sought in the coordinates stress vs strain. In this case, t is a parameter. This approach was proposed by Rabotnov in [1]. If isochronic creep curves are similar, then the relationship between the stresses, strains, and time can be represented as the product of two functions [1]:

$$\sigma=\varphi(\varepsilon)\psi(t), \tag{1.2}$$

where the similarity coefficient λ is given by the relation [7] $\lambda_k=\varphi(\varepsilon)/\sigma_k$ (ε is the strain, including the elastic component; σ_k is the coordinate of the isochrone at $t = t_k$ ($k = 0, 1, \ldots, n$)).

For the function $\psi(t)$, good results are given by the approximation [1, 8-11]

$$\psi(t)=1/(1+at^b), \tag{1.3}$$

where a and b are empirical coefficients ($a > 0$, $b \le 1$). At $t = 0$, the function $\psi(0) = 1$, while $\varphi(\varepsilon)$ is the same as the function which gives the instantaneous stress-strain curve.

Similarity condition (1.2) for transient creep has been validated experimentally for

Kiev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 166-172, July-August, 1991. Original article submitted June 13, 1989; revision submitted March 20, 1990.

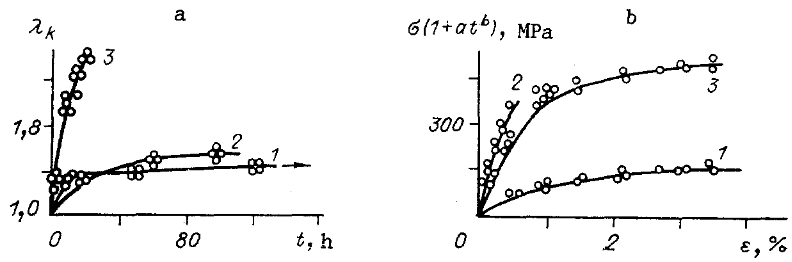

Fig. 1

nearly an infinite base for a wide range of materials [1, 7, 9]. We will additionally evaluate the possibility of generalizing Eq. (1.2) to all characteristic stages of creep, including the third stage. We also choose the approximating function for $\psi(t)$ in the form (1.3), while we determine the coefficients a and b by the method proposed in [10, 11]. We evaluate the validity of criterion (1.2) on the basis of two conditions: obtaining a single dependence of λ on t with variation of the stresses; reducing the family of isochrones, including the instantaneous stress-strain curve, to a single curve in the coordinates $\sigma(1 + at^b)$ vs ε. Some of the results are shown in Fig. 1 a and b (line 1 is for glass-textolite at T = 20°C [8], line 2 is for steel ÉI481 at T = 700°C, and line 3 is for alloy VZhL12U at T = 1000°C). It can be seen that the experimental points are grouped fairly closely (error no greater than 12-15%) around single curves with a change in stress from 0.2 to $0.8\sigma_Y$ within the range from 1 to 1000 h. Creep curves including the third stage were typically used in the analysis, the duration of this stage amounting in some cases to 70% of the time to rupture.

2. Unidimensional Constitutive Equations. Similarity condition (1.2) for isochronic creep curves was established more than 40 years ago and has been substantiated for a broad range of materials, but it has not been used to construct constitutive equations. We will examine one possible method of solving this problem, limiting ourselves in the first step to a unidimensional formulation.

With allowance for (1.3), we write similarity condition (1.2), which includes the instantaneous stress-strain curve $\varphi_0(\varepsilon^s)$ as the isochrone at t = 0, in the form of the relation

$$\varphi_0(\varepsilon^s) = \varphi_t(\varepsilon^r)[1 + G(t)]. \tag{2.1}$$

With $\varepsilon^s = \varepsilon^r$ and $\varphi_t(\varepsilon^r) = \sigma_0$, this relation becomes

$$\varphi_0(\varepsilon^r) = \sigma_0[1 + G(t)] \tag{2.2}$$

and we postulate that the instantaneous nonlinearity is identical in character to the creep nonlinearity. Here, ε^s is the non-viscous strain; $\varepsilon^r = \varepsilon^0 + \varepsilon^c$ is the viscous strain, including the initial strain ε^0 and the creep strain ε^c; φ_t is a function of this strain. In (2.1) we also took $G(t) = at^b$.

The main idea behind representation (2.2) is that the creep process in the plane φ_0, ε develops in a manner similar to instantaneous deformation and is completely determined by the form of the function φ_0 (Fig. 2a). In the general case, the deformation process can be regarded as an alternation of active ($d\sigma > 0$, $dt = 0$) and passive ($d\sigma = 0$, $dt > 0$) regimes of variation of the function $\varphi_0(\varepsilon^s)$ (due to an increment in the load or a time increment). Thus, loading (t = 0) to the prescribed stress σ_0 (point A in Fig. 2a) corresponds to motion along the straight line OA and causes the deformation $\varepsilon^0 = \sigma_0/E$. After a certain time interval $t(\sigma_0 = \text{const})$, creep strain ε^c accumulates and point A is shifted to position C.

Differentiating both sides of (2.2) with respect to t, we obtain the relation

$$\frac{d\varphi_0(\varepsilon^r)}{d\varepsilon^r}\frac{d\varepsilon^r}{dt} = \frac{d}{dt}\{\sigma_0[1 + G(t)]\}. \tag{2.3}$$

Solving this relation for the rate of accumulation of viscous strain, we find the creep equation in the form

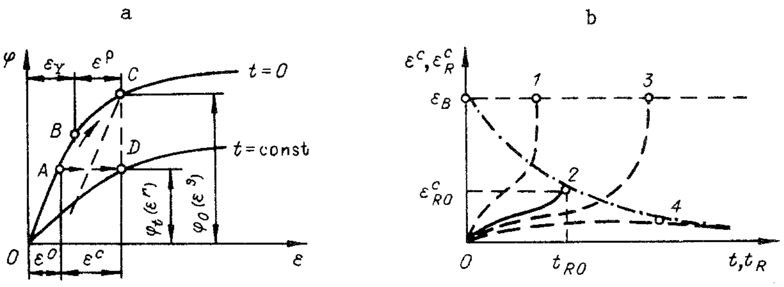

Fig. 2

$$\frac{d\varepsilon^r}{dt}=\frac{d}{dt}\{\sigma_0[1+G(t)]\}\left[\frac{d\varphi_0(\varepsilon^r)}{d\varepsilon^r}\right]^{-1}, \tag{2.4}$$

where $d\varphi_0(\varepsilon^r)/d\varepsilon^r = g_0(\varepsilon^s)$ characterizes the rate of strain-hardening of the medium. It is obvious that, within the framework of constitutive equation (2.4), the character of creep will be determined by the structure of the time operator G(t) and the form of the instantaneous stress-strain curve $\varphi_0(\varepsilon^s)$.

We give the instantaneous stress-strain curve $\varphi_0(\varepsilon^s)$ in the form ofthe system of equations

$$\varphi_0(\varepsilon^s)=\begin{cases}E\varepsilon^e & \text{at} \quad 0\leqslant\varepsilon^s\leqslant\varepsilon_Y,\\ E\varepsilon_Y+\varphi_0(\varepsilon^p) & \text{at} \quad \varepsilon_Y\leqslant\varepsilon^s\leqslant\varepsilon_B\end{cases} \tag{2.5}$$

and as an example examine features of the creep of materials which undergo linear ($\varphi_0(\varepsilon_p) = E^*\varepsilon^p$) and power-law ($\varphi_0(\varepsilon^p) = B(\varepsilon^p)^m$) strain-hardening. Here, ε^e and ε^p are the elastic and plastic components of the non-viscous strain; ε_Y and ε_B are the strains corresponding to the yield point and ultimate strength; E is the elastic modulus; E^* is the linear strain-hardening modulus; B and m are power strain-hardening coefficients.

First of all we note that in the strain region $\varepsilon^r < \varepsilon_Y$ creep is independent of the character of the initial strain-hardening of the material since $g_0(\varepsilon^s) \equiv E$ for all materials. For the creep rate, we can use (2.4) to write

$$\frac{d\varepsilon^c}{dt}=ab\frac{\sigma_0}{E}\frac{1}{t^{1-b}}, \tag{2.6}$$

where we have taken $\varepsilon^c = \varepsilon^r - \varepsilon^0$; $G(t) = at^b$; $g_0(\varepsilon^s) = E$.

In the strain region $\varepsilon^r > \varepsilon_Y$ for materials exhibiting linear strain-hardening, we find from (2.4) that

$$\frac{d\varepsilon^c}{dt}=ab\frac{\sigma_0}{E^*}\frac{1}{t^{1-b}}, \tag{2.7}$$

while for materials exhibiting power-law strain-hardening

$$\frac{d\varepsilon^c}{dt}=ab\left(\frac{1}{B}\right)^{1/m}\frac{\sigma_0}{m}\left[\sigma_0(1+at^b)-E\varepsilon_Y\right]^{(1-m)/m}\frac{1}{t^{1-b}}. \tag{2.8}$$

Here, $g_0(\varepsilon^s) = E^*$ and $g_0(\varepsilon^s) = Bm(\varepsilon^r)^{m-1}$, respectively. Figure 3 presents a graphical interpretation of the features of creep established by Eqs. (2.6)-(2.8).

It can be seen that in the strain region $\varepsilon^c < \varepsilon_Y - \varepsilon^0$ (curves 1) and for materials with linear strain-hardening (curves 2 and 3 in Fig. 3a), creep is transient ($\dot\varepsilon^c \to 0$ with $t \to \infty$ and $g_0(\varepsilon^s) = \text{const}$) and the creep curve has no points of inflection since the second derivative of ε^c with respect to t

$$\frac{d^2\varepsilon^c}{dt^2}=ab(b-1)\frac{\sigma_0}{g_0(\varepsilon^s)}\frac{1}{t^{2-b}}$$

is always less than zero (b - 1 < 0).

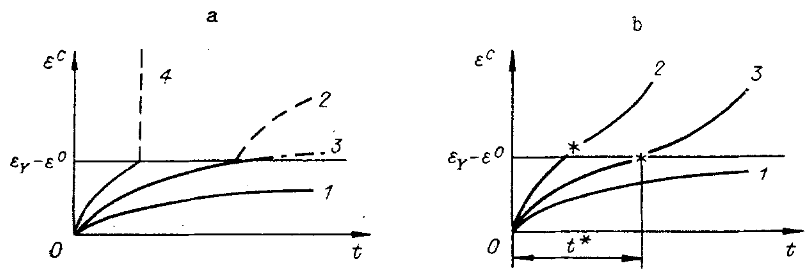

Fig. 3

In materials which undergo a pronounced transition from an elastic section to a section with linear strain-hardening, there is a sudden increase in the rate of transient creep (curve 2 in Fig. 2b) at a constant stress. This effect is connected with a sudden change in the strain-hardening modulus $g_0(\varepsilon^s)$ by the amount $\Delta g_0(\varepsilon^s) = E - E^*$ as the viscous strain ε^r reaches ε_Y at the moment of time when $\psi_0(\varepsilon^r) = \varphi_0(\varepsilon_Y)$. This phenomenon is demonstrated to exist in Part 3, with the use of a heat-resistant material as an example.

The creep of materials which undergo strain-hardening includes all three stages (curves 2 and 3 in Fig. 3b). In this case, the second derivative of ε^c with respect to t

$$\frac{d^2\varepsilon^c}{dt^2} = ab\left(\frac{1}{B}\right)^{1/m}\frac{\sigma_0}{m}\left[\sigma_0(1+at^b)-E\varepsilon_Y\right]^{(1-m)/m}\left\{\frac{1-m}{m}\,\frac{ab\sigma t^b}{\left[\sigma_0(1+at^b)-E\varepsilon_Y\right]}-(1-b)\right\}\frac{1}{t^{2-b}}$$

at the moment of time

$$t^* = \left[\frac{1-b}{a\left(\frac{b}{m}-1\right)}\left(1-\frac{\sigma_Y}{\sigma_0}\right)\right]^{1/b} \qquad (2.9)$$

becomes equal to zero and a point of inflection (denoted by the asterisk in Fig. 3b) appears on the creep curve. Creep rate increases after the point of inflection, since, in accordance with (2.4), at $g_0(\varepsilon^r) \to 0$ $\dot{\varepsilon}^r \to \infty$. The section of the creep curve near the inflection point is close to linear and corresponds to steady-state creep. In the case when $g_0(\varepsilon^r) = 0$, creep occurring at $\varepsilon^c > \varepsilon_Y - \varepsilon^0$ is nearly independent of time (curve 4 in Fig. 3a).

3. Prediction of Creep. In creep problems, it is important to predict and calculate plastic strains occurring during steady, stepped, and cyclic loading regimes. To do this within the framework of the above constitutive relations, we must have the function $\varphi_0(\varepsilon^s)$ and the values of the rheological constants a and b.

The function $\varphi_0(\varepsilon^s)$, including the values of E, E^*, B, m, σ_Y, ε_Y, ε_B, is assigned on the basis of approximation of experimental data from the uniaxial tension of smooth cylindrical specimens. With the use of the same tension curve and one creep curve, the coefficients a and b are determined from the relation [10]

$$\lg a + b \lg t_k = \lg(\sigma_k/\sigma - 1) \quad (k = 1, 2, \ldots, i), \qquad (3.1)$$

where t_k is an arbitrary moment of time on the base creep curve; σ_k is the stress on the tension curve corresponding to the creep strain at the moment of time t_k; the angle of inclination of straight line (3.1) corresponds to the value of b; the segment intercepted on the y axis ($t_k = 1$) is equal to log a. Table 1 shows values of the coefficients calculated for certain structural materials. The base creep curve was limited to the steady stage.

First we will examine a steady (σ_0 = const) loading regime. In this case, to calculate the creep strains it is sufficient to use equations that can be solved relative to ε^c. Then integrating (2.6) and (2.7) with the initial condition $\varepsilon^c = 0$ at $t = 0$ for materials with linear strain-hardening, we obtain

$$\varepsilon^c = \begin{cases} \frac{a}{E}\sigma_0 t^b, \\ \frac{\sigma_0}{E}(1+at^b) + \frac{\sigma_Y}{E}\left(1-\frac{E}{E^*}\right) - \frac{\sigma_0}{E}, \end{cases} \qquad (3.2)$$

TABLE 1

Alloy	T, °C	a,$(1/n)^b$	b	$E\cdot 10^{-2}$, MPa	$E^*\cdot 10^{-2}$, MPa	m	$B\cdot 10^{-2}$, MPa	σ_Y, MPa
12Kh1MF	540	0,005	0,95	712	—	0,30	6,5	285
ÉI481	600	0,059	0,33	900	28,0	—	—	450
ÉI929VD	800	0,087	0,34	653	—	0,28	94,0	620
	650	0,048	0,30	1150	80,2	—	—	550
ÉI437B	800	0,105	0,60	720	—	0,33	110,0	360
	850	0,100	0,77	870	—	—	—	310
ÉI867	900	0,078	0,55	745	—	0,41	82,0	430
	950	0,407	0,56	680	—	0,41	82,0	320
VZhL12UU	1000	0,269	0,44	440	—	0,32	81,0	375

while integration of (2.6) and (2.8) for materials with power-law strain-hardening yields

$$\varepsilon^c = \begin{cases} \dfrac{a}{E}\sigma_0 t^b, \\ \left(\dfrac{1}{B}\right)^{1/m}\left[\sigma_0(1+at^b)-\sigma_Y\right]^{1/m}+\dfrac{\sigma_Y-\sigma_0}{E}. \end{cases} \tag{3.3}$$

The results of calculations (lines) performed with Eqs. (3.2) and (3.3) are compared in Fig. 4 with experimental data (points) for alloy ÉI437B (a) at T = 650°C and σ = 400, 470, and 500 MPa (lines 1-3) and steel 12Kh1MF (b) at T = 540°C and σ = 200, 250, and 285 MPa (lines 1-3). The upper right-hand corner shows the character of instantaneous deformation. The position of the point of inflection calculated from Eq. (2.9) is denoted by an asterisk. The experimental data for alloy ÉI437B was borrowed from [8].

Creep occurring at stresses which vary over time is calculated with the use of constitutive equations of the rate type (2.6)-(2.8). These equations in essence reflect time-dependent strain-hardening. We will examine the case of a single stepped stress change in a regime which involves additional loading and partial unloading. The results of the calculation are shown in Fig. 4c along with experimental curves (points) for alloy ÉI437B at T = 70°C, σ_1 = 300 → σ_2 = 400 MPa and T = 800°C, σ_1 = 300 → σ_2 = 200 MPa (lines 1 and 2). The moments of additional loading and partial unloading are denoted by the dark points on curves 1 and 2.

We assign the regime of cyclic loading with the condition

$$\sigma_0 = \sigma_m + \sigma_a\gamma(f,\ t), \tag{3.4}$$

where σ_m and σ_a are the static and cyclic components of the stresses; γ is a periodic function of time t and frequency f. As is known [1, 2], in actual materials, a cyclic load (3.4) speeds up or slows down creep compared to steady (σ_a = 0) loading.

The effect of cyclic loads on the creep process can be accounted for in the form of an alternation of additional loadings and unloadings, with the use of (2.6) and (2.8). In the case of rapid cyclic loading (f > 10 Hz), it is sufficient to consider the effect of σ_a on the yield point σ_Y. This effect is equivalent to an equidistant displacement of the function $\varphi_0(\varepsilon^p)$ in (2.5) by the amount σ_a [12]. The cyclic load in this case obviously turns out to have an effect on creep in the strain range $\varepsilon^c > \varepsilon_Y - \varepsilon^0$.

Let us examine the case when $\varphi_0(\varepsilon^p)$ decreases by the amount σ_a. Then, using (3.2) for the case of linear strain-hardening, we obtain the equation governing cyclic creep in the form

$$\varepsilon_a^c = \frac{\left[\sigma_m(1+at^b)+\sigma_a\right]}{E^*} + \frac{\sigma_Y}{E}\left(1-\frac{E}{E^*}\right) - \frac{\sigma_m}{E},$$

while the use of (3.3) for power-law strain-hardening yields

$$\varepsilon_a^c = \left(\frac{1}{B}\right)^{1/m}\left[\sigma_m(1+at^b)-(\sigma_Y-\sigma_a)\right]^{1/m} + \frac{(\sigma_Y-\sigma_m)}{E}, \tag{3.5}$$

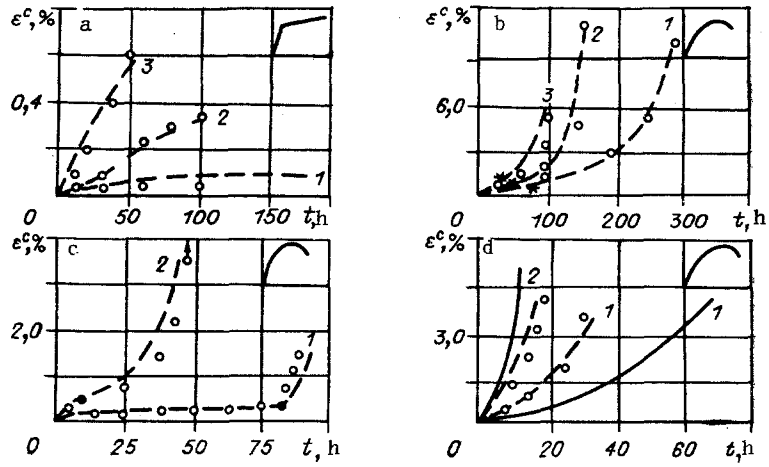

Fig. 4

It follows from this that a cyclic load accelerates the creep process. Here, ε_a is the cyclic creep strain, while $\sigma_m = \sigma_0$.

A strain-hardening effect (an increase in $\varphi_0(\varepsilon^p)$) during cyclic loading is seen in materials which undergo power-law strain-hardening [12]. In the given case, by analogy with (3.5)

$$\varepsilon_a^c = \left(\frac{1}{B}\right)^{1/m}\left[\sigma_m\left(1+at^b\right) - (\sigma_Y+\sigma_a)\right]^{1/m} + \frac{(\sigma_Y-\sigma_m)}{E} \tag{3.6}$$

and the creep lag effect takes place.

Figure 4d compares results calculated from Eqs. (3.5) and (3.6) with experimental results (points) for alloy ÉI867 with $\sigma_m = 200$ and $\sigma_a = 50$ MPa, respectively, for T = 900 and 950°C (lines 1 and 2). The solid lines represent static ($\sigma_a = 0$) creep curves.

On the whole, it can be seen from Fig. 4 that the agreement between the theoretical and experimental results is quite satisfactory. In particular, it is seen for materials which undergo linear strain-hardening that only transient creep (a) can occur, while all three stages of creep can take place for materials which exhibit power-law strain-hardening (b).

4. Prediction of Rupture Strength. We determine the life of a material under creep conditions for a specified stress σ_k as the point on the creep curve whose abscissa corresponds to the moment of fracture t_{Rk} and whose ordinate corresponds to the creep strain accumulated up to this moment ε_{Rk}^c (see Fig. 2b). In this case, rupture strength can be calculated (with variation of σ_k) by simultaneously solving the system of equations

$$\varepsilon^c = \int_0^t f\left[\sigma_k, \varphi_0, (\varepsilon^s), G(t)\right] dt, \quad \varepsilon_{Rk}^c = \Psi(t_{Rk}), \tag{4.1}$$

giving the creep law and the fracture criterion, respectively.

The creep law can be chosen in the form (3.2) or (3.3), depending on the character of the initial strain-hardening of the material. We choose the function Ψ by using a representation on fracture under creep conditions as a process which is accompanied by embrittlement of the material [1-4]. Within the framework of Eqs. (2.4) and (2.5), the maximum creep strain accumulated by the moment of fracture ε_R^c will not exceed ε_B (see points 1 and 2 in Fig. 2b). We then define the embrittlement as the process of a reduction in ε_R^c in relation to ε_B. For the function $\Psi(t_R)$, we chose an exponential law (points 2 and 4 in Fig. 2b) such that

$$\varepsilon_R^c = \varepsilon_B \exp\left(-\frac{t_R}{L}\right) = \varepsilon_B\left(\frac{\varepsilon_{R0}^c}{\varepsilon_B}\right)^{-t_R/t_{R0}}. \tag{4.2}$$

Here, L is an exponent; ε_{R0}^c, t_{R0} are the strains accumulated by the moment of fracture and the time to fracture determined from the base creep curve (curve 2 in Fig. 2b). It was assumed in (4.2) that at $t_R = 0$ $\varepsilon_R^c \equiv \varepsilon_R$, at $t_R > 0$ $\varepsilon_R^c < \varepsilon_B$, and at $t_{R4} > t_{R2}$ $\varepsilon_{R4} < \varepsilon_{R2}$.

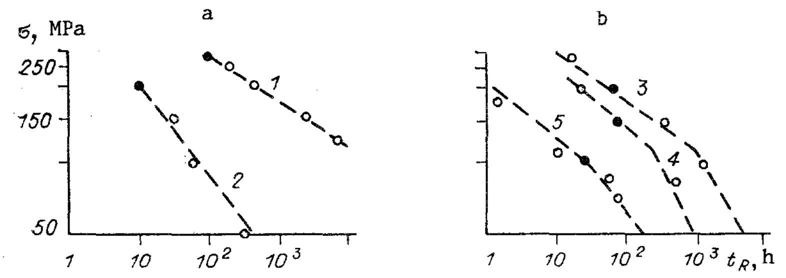

Fig. 5

Exponential relation (4.2) should be interpreted not as a physical law, but as an approximation which is convenient for determining the coefficients.

One feature of the proposed approach is the possibility of analytically determining the points of inflection on the rupture-strength curve. This possibility exists because the creep law in (4.1) is given by system (3.2) or (3.3), and the point of inflection will be determined by the attainment of the creep strain ε_Y. In particular, for materials which undergo power-law strain-hardening, the time to rupture t_R^*, corresponding to the point of inflection on the rupture-strength curve, is found from the relation

$$t_R^* = \left(\frac{E\varepsilon_Y - \sigma^*}{a\sigma^*}\right)^{1/b} = \left(\frac{\sigma_Y - \sigma^*}{a\sigma^*}\right)^{1/b}, \tag{4.3}$$

where σ^* is the stress at which the creep strain $\varepsilon^c = \varepsilon_Y$ is accumulated during the time t_R.

Figure 5 shows some predictions of rupture strength in comparison with experimental data (points) for chromium-molybdenum steel 12Kh1MF at T = 540°C and heat-resistant nickel alloys ÉI437B (at T = 850 and 800°C), VZhL12U (at T = 1000°C), and ÉI867 (at T = 1000°C (lines 1-5)). The calculation was performed using Eqs. (3.3) and (4.2). Curves 1 and 2 were calculated on the basis of the equation $\varepsilon_R^c \equiv \varepsilon_B$. The points of inflection on rupture-strength curves 2-4 were calculated from Eq. (4.3). The base creep curves are represented by the dark points. The maximum difference between the theory and experiment is 30% and is seen mainly in the high-stress region. In problems of practical importance involving the prediction of rupture strength for a base commensurate with the actual service life, the difference between theory and experiment is no greater than 20%.

LITERATURE CITED

1. Yu. N. Rabotnov, Creep of Structural Elements [in Russian], Nauka, Moscow (1966).
2. V. V. Moskvitin, Cyclic Loads on Structural Elements [in Russian], Nauka, Moscow (1981).
3. S. A. Shesterikov, Laws of Creep and Rupture Strength [in Russian], Mashinostroenie, Moscow (1983).
4. J. T. Boyle and J. Spense, Stress Analysis for Creep, Butterworths, Woburn, MA (1983).
5. O. V. Sosnin, B. V. Gorev, and A. F. Nikitenko, Energy Variant of Creep Theory [in Russian], IG SO AN SSSR, Novosibirsk (1986).
6. C. C. Davenport, "Correlation of creep and relaxation properties of copper," J. Appl. Mech., 5, No. 2 (1938).
7. M. A. Koltunov, Creep and Relaxation [in Russian], Vysshaya Shkola, Moscow (1976).
8. I. P. Bulygin, P. T. Vlasova, A. T. Gorbodei, et al., Atlas of High-Temperature Creep and Stress-Rupture Curves of Steels and Alloys for Engines [in Russian], Oborongiz, Moscow (1957).
9. R. M. Goldhoff, "The application of Robotnov's creep parameter," Proc. ASTM, 61, 907 (1961).
10. Yu. N. Rabotnov and S. T. Mileiko, Transient Creep [in Russian], Nauka, Moscow (1970).
11. V. P. Golub and A. S. Oleinik, "Method of calculating coefficients in the Rabotnov creep parameter," Mashinovedenie, No. 2 (1988).

12. V. P. Golub, A. S. Oleinik, and V. N. Pavlov, "Features of short-term deformation during vibration," Mashinovedenie, No. 5 (1988).

ENERGY VARIANT OF THE UNIAXIAL THEORY OF CREEP AND RUPTURE STRENGTH

V. P. Radchenko

UDC 539.376

All of the procedural recommendations contained in [1] which pertain to the third stage of creep are based either on strain-hardening theory or on flow theory, and they all have several shortcomings. One deficiency is the impossibility of describing reverse creep during unloading. Ignoring the latter in calculations will lead to errors in finding the time to rupture, particularly under transient and cyclic loads. Another unresolved problem is formulating the governing rheological equations, which make it possible to describe creep beyond the elastic limit. There is also the question of the selection of a fracture criterion that could be used to describe the following experimentally observed facts: the nonmonotonic character of the limiting inelastic strain during fracture; the nonlinear character of rupture-strength curves; the presence of a stage of "avalanche" creep. Thus, in the present study, we want to develop a creep theory and fracture criterion for metals that will allow us to solve the problems just mentioned.

1. We used the method of strain separation as the basis for construction of the corresponding rheological equations. This method has been proposed for the first and second stages of creep [2]. To decribe the third stage, it is customary to adopt a hypothesis in which the damage process is directly connected with the cumulative inelastic strain and the running stress. One characteristic of the state of the material is the damage parameter, which is linked with the relative reduction in the cross-sectional area of the specimen and the consequent increase in the true stress due to microscopic fracture of the material during deformation [3-9].

In the present study, we further develop the energy approach proposed in [10-12] to describe the stage of softening of the material. In accordance with this approach, the damage parameter is assumed to be proportional to the linear combination of the amounts of work done by the true stress on creep strain and on plastic deformation. The main form of the governing equations is as follows

$$\varepsilon = e + e^p + p,\ \dot{e} = \dot{\sigma}/E,\ \dot{e}^p = \varkappa S'(\sigma)\dot{\sigma},\ p = u + v + w,$$

$$u(t) = \sum_{k=1}^{s} u_k(t),\ \dot{u}_k(t) = \lambda_k\left[a_k(\sigma(t)/\sigma_*)^n - u_k(t)\right],$$

$$v(t) = \sum_{k=1}^{s} v_k(t),\ \dot{v}_k(t) = \begin{cases}\lambda_k\left[b_k(\sigma(t)/\sigma_*)^n - v_k(t)\right],\ b_k(\sigma(t)/\sigma_*)^n > v_k(t),\\ 0,\ b_k(\sigma(t)/\sigma_*)^n \leqslant v_k(t);\end{cases}$$

$$\dot{w}(t) = c\left(\sigma(t)/\sigma_*\right)^m; \qquad (1.1)$$

$$\sigma = \sigma_0(1 + \omega); \qquad (1.2)$$

$$\dot{\omega} = \gamma\sigma\dot{e}^p + \alpha\sigma\dot{p}, \qquad (1.3)$$

where ε is the total strain; e and e^p are the elastic and plastic strain; p is the creep strain; u, v, and w are the viscoelastic, viscoplastic, and viscous components of p; σ_0 and σ are the nominal and true stresses; E is the Young's modulus; λ_k, a_k, b_k, c, n, m, σ_* are rheological constants of the material which can be used to describe the first and

---

Kuibyshev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 172-179, July-August, 1991. Original article submitted March 13, 1990.

